Anthropologica et Præhistorica, 117, 2006, 135 – 162

Mathematics in (central) Africa before colonization

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Summary

The paper provides a summary of (black) African ethnomathematics, with a special focus on results of possible interest to eventual mathematical properties of the Ishango rod(s). The African diversity in number names, gestures and systems (including base 2 of the Bushmen, probably related to the early Ishango people) shows frequent decompositions of numbers in small groups (similar to the carvings on the rod), while the existence of words for large numbers illustrates counting was not merely done for practical reasons. A particular case is the base 12, with it straightforward finger counting method on the hands, and used in Nigeria, Egypt and the Ishango region. Geometric representations are found in traditional sand drawings or decorations, where lines and figures obey abstract rules. Number lines drawn in the sand (using small and long lines as on the rod) make anyone immediately remind the Ishango carvings. Knotted strings and carved counting sticks (even looking like exact wooden copies of the Ishango rod) illustrate an African counting practice, as confirmed in written sources of, for instance, a gifted American slave. Finally, mancala mind games, Yoruba and Egyptian multiplication (using doublings as on the Ishango rod) or kinship studies all show ethnologists may have ignored for too long Africans were talking the mathematical language, ever since.

Samenvatting

Het artikel geeft een samenvatting van (zwarte) Afrikaanse etnowiskunde, met een bijzondere aandacht voor resultaten met een mogelijk belang voor de wiskundige eigenschappen van de Ishangobeentje(s). De Afrikaanse verscheidenheid in getallennamen, -gebaren en -systemen (met inbegrip van de basis 2 van de Bushmannen, waarschijnlijk verwant aan de eerste Ishangovolkeren) toont de frequente ontbindingen van getallen in kleine groepen (zoals op het Ishangobeen), terwijl het bestaan van woorden voor grote getallen illustreert dat tellen niet alleen voor praktische redenen gebeurde. Een bijzonder geval is de basis 12, met haar evidente telmethode op de vingers van de hand, en die gebruikt werd in Nigeria, Egypte en het Ishangogebied. Meetkundige voorstellingen worden teruggevonden in traditionele zandtekeningen of decoraties, waarin de lijnen en figuren gehoorzamen aan abstracte regels. Getallenstrepen getekend in het zand (met korte en lange strepen zoals op het been) brengen bij eenieder onmiddellijk het Ishangobeen voor de geest. Geknoopte koorden en gekerfde stokken (die zelfs lijken op exacte houten kopieën van het Ishangobeen) illustreren een Afrikaanse telpraktijk, zoals die bevestigd werd in geschreven bronnen over, bijvoorbeeld, een begaafde Amerikaanse slaaf. Tenslotte tonen de mancaladenkspelen, de Yorubese en Egyptisch vermenigvuldiging (die verdubbelingen gebruiken zoals op het Ishangobeen) of de studies van familiebanden allen aan dat etnologen te lang ontkend hebben dat Afrikanen reeds erg lang de taal van de wiskunde spraken.

1. INTRODUCTION

1.1. Rationale

In this journal addressed mainly to archaeologists, historians and anthropologist, a survey of today's state of the art on (ethno)-mathematics written by a mathematician may be appropriate. Indeed, even in the 21st century, the prejudice persists mathematical activity was completely lacking in Africa, despite the many publications, conferences and lectures on the topic. However, (ethno)-mathematics hardly gets an opportunity to infiltrate into protectionist human sciences and thus non-mathematical circles may still doubt the mathematical community considers, for instance, the Ishango rod as a "true" testimony of a mathematical activity.

Although the present paper addresses to non-mathematicians, there will be a few specialist mathematical passages, where the non-mathematical reader can safely shut the eye, but these notions were maintained in the hope of gaining some esteem for these African mathematics and showing they are more than recreational mathematics. The basic goal is shaping the mathematical context of the Ishango rod and thus topics from the region of Congo, Rwanda and Burundi, were preferred from the vast range of ethnomatematical examples (Huylebrouck, 2005). Finally, note this focus on

African realizations does not imply an intention to replace a criticized euro-centrism by an equally disprovable afro-centrism. The "mathematical Out of Africa" debate and the "Black Athena Debate", about the black-African influence on Greek and Western culture, are discussions the author happily leaves to others.

Ethnomathematics concentrates on the importance of native culture for mathematics (Nelson, 1993). Though mathematics is a universal science by excellence, learning it is another matter: imagine two students learn a foreign language. The first uses the scholarly method of grammatical rules with declinations and conjugations. The second learns the language through the approach of repeating short sentences, with or without audiocassette. After a while, both understand the same language, but they will seldom have an identical thorough command and linguistic feeling. For mathematics, or rather, the teaching of mathematics, the situation is comparable.

There is for instance one of the first ethnomathematical studies, done by two Americans, J. Gay and M. Cole, when they were Peace Corps volunteers (Gay, 1967). They wondered if there were any appreciable differences between the mathematical skills of American and African students. Gay and Cole let Kpelle (a people living in Liberia) and Americans undergo several experiments, obtaining statistical data on the differences and similarities in quantitative skills such as estimating volumes or distances, the measurement of time, *etcetera*. Figure 1 shows the example in which Gay and Cole studied estimations of time intervals of 15, 30, 45, 60, 75, 90, 105, and 120 seconds, and the test showed that there was no noticeable difference between both groups for the estimation of longer periods.

1.2. Sources for ethnomathematics

Information about African ethnomathematics is collected in four ways: there are (1) written sources in Egyptian temples, (classical) Greek texts and some scarce reports on American slaves, (2) oral chronicles, (3) recent observations of traditional customs, and (4) the archaeological findings. We give a survey of their importance:

(1) The accuracy of some ancient written sources can be surprising. For instance, there are the maps based on Ptolemy that use the reference "Lunae Montes" or "Mountains of the Moon", to designate the region of the sources of

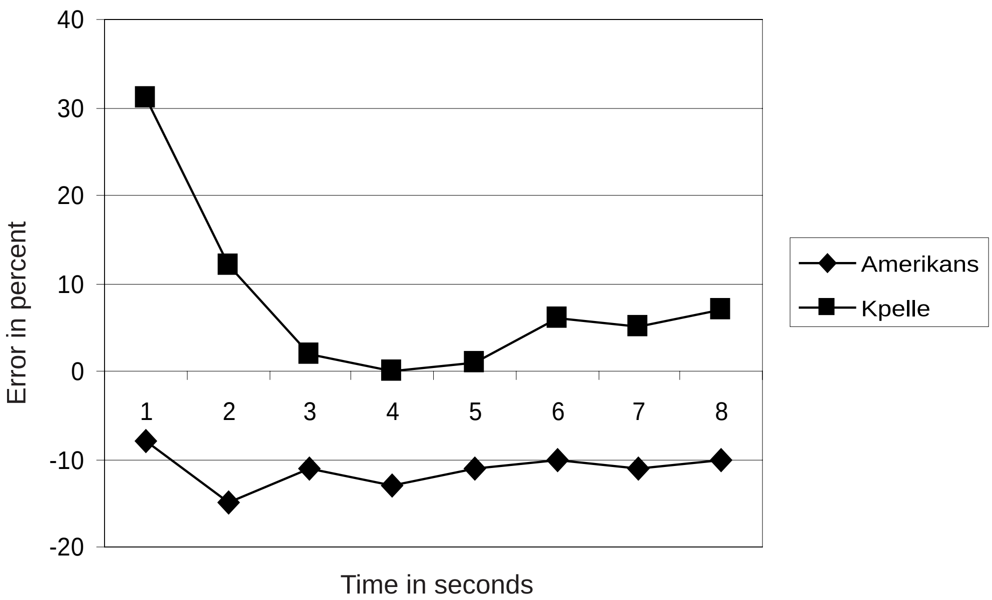


Fig. 1 — Time estimates by Kpelle student and by Americans (Gay & Cole, 1967).

the Nile (sometimes adding a region of one-eyed people as well). Roman legends speak about Pygmy people living at these "Montes Lunae", while Egyptian texts refer to "little men of the forest and land of the spirits". Arab tales situate the Biblical Gardens of Eden in this "Jebel Kamar" region, actually called "Unyamwezi" ("mwezi" = "moon").

(2) The tradition of long recitations, from generation to generation, constitute true oral testimonies of history, though extreme care is mandatory, such as with the Dogon story who would have known Sirius is a double star. However, when solar eclipses are mentioned in different regions and by different narrators, they provide an accurate dating when they confirm each other, independently.

(3) Traditional forms of living observed by the first travelers seldom mention mathematical topics. For sure, they were not in the principal fields of interest of explorers and missionaries. However, from the descriptions of counting practices, games, or designs, some information can be recovered.

(4) Archaeological findings, such as excavated teeth or bones, sometimes show patterns (d'Errico, 2001). Again, the importance of these "proto-mathematics" has to be evaluated judiciously. For example, there is 30,000 years old carved flat stone, found in Blanchard in the French Dordogne, in which some discover phases of the moon; or a Kenyan megalithic site of about 300 B. C. with 19 pillars said to be oriented along heavenly constellation; or bird statues in the walled town of Zimbabwe along a Southern Cross pattern. Common interpretations of these findings have some new age tendencies and need further investigation.

2. Creative counting in Africa

2.1. Number bases

Several authors wrote about the beginning of counting when naming numbers often was the only "mathematical" operation. During the first steps, elementary arithmetic was scarce, and in many cases, the latter even is an overstatement (Delafosse, 1928; Moiso, 1985; 1991). Figure 2 shows two maps, one following G. G. Joseph (Joseph, 1992), with basis 2, 4, 20 and

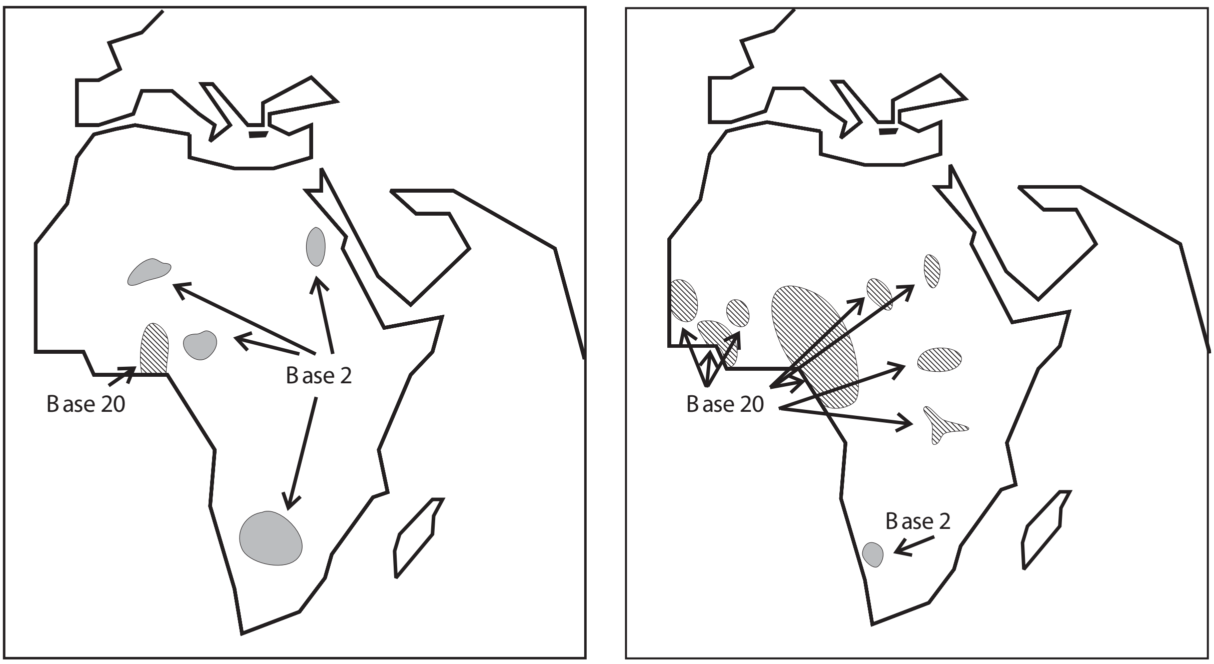


Fig. 2 — (Non-exhaustive) maps of number systems, by Joseph (left; Joseph, 1992) and by Barrow (right; Barrow, 1993).

10 systems, and one following John D. Barrow, showing 20 and 10 systems (Barrow, 1993).

The simplest number systems from regions in Central-Africa and South-America are simple enumerations: *one*, *two*, *two-and-one*, *two-two*, *many*. For instance, for the Gumulgal of Australia:

1 = urapon; 2 = ukusar; 3 = ukusar-urapon; 4 = ukusar-ukusar; 5 = ukusar-ukusar-urapon; 6 = ukusar-ukusar-ukusar.

Note these systems are not binary in the mathematical sense, as 4 is not necessarily 2^2, nor is $8 = 2^3$. Similar counting methods seem to stop at 6, though some authors put three dots at the end; does this imply they counted to, say, a thousand? However, the counting method of the Bushmen, who may play a role in the Ishango story, goes as far as 2+2+2+2+2:

1 = xa; 2 = t'oa; 3 = 'quo; 4 = t'oa-t'oa; 5 = t'oa-t'oa-xa; 6 = t'oa-t'oa-t'oa; 7 = t'oa-t'oa-t'oa-xa; 8 = t'oa-t'oa-t'oa-t'oa …

The use of 2 sometimes led to the following notations for 6, 7 and 8:

6: | | | 7: | | | | 8: | | | |
   | | | | | | | | | |

Reading these arrangements horizontally, they yield the combinations 3 + 3, 4 + 3 and 4 + 4. The expressions illustrate a first evolution. For instance, the Mbai use such an additive system: *6 = muta muta* or: *3+3*; *8 = soso* or: *4+4*; *9 = sa dio mi* or: *4+5*, while for the Sango from northern Congo *7 = na na-thatu* or: *4+3*; *8 = mnana* or: *4+4*, and *9 = sano na-na* or: *5+4*. The later popularity of this "additive" method for the numbers 6 to 9 may have been due to its use in mental calculations. For example, to get the double of 7, 7+7 = (4+3) + (4+3), as 4+3+3 = 10, and thus the answer is 10+4. South of the Sahara, mental calculation was a tradition for centuries and traditional (mental) calculating techniques were indeed based on repeated doublings (see below).

2.2. Creativity in counting

A small number base has some advantages, since if for instance 5 is the base, 7 plus 8 becomes: "5 + 2" plus "5 + 3", and 2 + 3 = 5 changes the operation easily into 5 + 5 + 5 or 10 + 5. Thus, the Makoua from Northern Mozambique say 6 = *thanu na moza* or *5 + 1*; 7 = *thanu na pili* or *5 + 2*, while for the Bété in Ivory Coast 56 = *golosso-ya-kogbo-gbeplo*, or *20 times 2 and 10 and 5 and 1*. The Bulanda in West Africa use a base 6 system: 7 is *6 + 1*, 8 is *6 + 2*.

The Yasayama from Congo use base 5 (Maes, 1934):

1 = omoko; 2 = bafe; 3 = basasu; 4 = bane; 5= lioke; 6 = lioke lomoko; 7 = lioke lafe; 8 = lioke lasasu; 9 = lioke lane; 10 = bokama; 11 = bokama lomoko; 12 = bokama lafe ...

It is not a true base 5 system, because 25 = 5×5 plays no particular role, but Congolese Baali system is more remarkable because 4×6 = 24 indeed plays the role of a base, since when $576 = 24^2$ is reached, a new word is invented:

1 = imoti; 2 = ibale; … 5 = boko; 6 = madia; 7 = madea neka (6 + 1)*; 8 = bapibale* (6 + 2)*; 9 = bap-ibale nemoti* (8 + nemoti = 8 + 1)*; 10 = bapibale nibale* (8 + nibale = 8 + 2)*; 11 = akomoboko na imoti* (10 + 1)*; 12 = komba; 13 = komba nimoti; 14 = komba nibale; … 24 = idingo; 25 = idingo nemoti; … 48 = modingo mabale; … 576 = modingo idingo* (= 24^2)*; 577 = modingo idingo nemoti* (= $24^2 + 1$) …

The Nyali from Central-Africa employ a mixed system using 4, 6 and 24 = 4×6:

1 = ingane; 2 = iwili; 3 = iletu; 4 = gena; 5 = boko; 6 = madea; 7 = mayeneka; 8 = bagena (= plural of four); … 24 = bwa; 576 = mabwabwa (= 24^2*), …*

Inhabitants of the same region, the Ndaaka, have 10 and 32 as base numbers. Thus, *10* is *"bokuboku"*, *12* is *"bokuboku no bepi"*, and for *32* there is a particular word, *"edi"*. Next *64* becomes *"edibepi"* (= *32*×2) while *1024* is *"edidi"* (or 32^2). A number such as 1025 is therefore expressed as *"edidi negana"* or $32^2 + 1$.

These differences are often surprising: in the tiny country of Guinea-Bissau, there are at least 4 different methods: the Bijago use a purely decimal system; the Manjaco a decimal system with the exceptions 7 = 6+1 and 9 = 8+1; the Balante mix bases 5 and 20; the Felup mix bases 10 and 20, with the exceptions 7 = 4 + 3 and 8 = 4 + 4. Counting words can differ, within a given language, depending on what has to be counted (people, objects, or animals). For instance, in Burundi, *6* can sometimes be *"itano n'umwe"* or *"5+1"*, but it can also become *"itandatu"*, or *3+3*. *"indwi"* or *7* can change in *"itano n' iwiri"* or *5+2*.

As the list of examples of this creativity seems boundless, let us finish by number words of the Huku-Walegga, from the region north-west of the down course of the Semliki River exactly at the shores of the river where the Ishango rod was found. They would express 7 as 6+1 and 8 as 2×4, while 16 would be (2×4)×2. Yet, the next three numbers are again formulated as sums, but 20 is 10×2. This surprising mathematical mixture is not so unexpected in view of the given examples.

2.3. Words for larger numbers

Maybe this counting creativity implied counting in Africa exceeded the purely practical applications, and also why some people went all the way to count large numbers tantalizing the imagination. In the language of Rwanda, *10,000* is *inzovu,* or: *an elephant*, and thus *20,000 = inzovu ebyilli*, that is, *two elephants.* Some say the language did not know any larger numerals (Coupez, 1960; Hurel, 1951 ; Rodegem, 1967), but Pauwels goes all the way to 100,000, while the Rwandan Abbé Kagame mentions *100,000 = akayovu* or *a small elephant*; *1,000,000 = agahumbi* or *a small thousand*; *10,000,000 = agahumbagiza* or *a small swarming thousand*; *100,000,000 = impyisi* or *a hyena*; *1,000,000,000 = urukwavu* or *a hare* (Kagame, 1960). The related language of Burundi also goes as far as *100,000 = ibihumbi ijana*, and in the Buganda kingdom, north of Rwanda, greater numerals existed too, such as 10,000,000. The Bangongo language, spoken in Congo, does not go as far:

100 = kama; 1,000 = lobombo;

10,000 = njuku; 100,000 = losenene.

The Tanzanian Ziba has a clear Swahili-influence:

100 = tsikumi; 1,000 = lukumi;

10,000 = kukumi.

The base 20 counters in Nigeria had a word for $20^4 = 160{,}000$: *"nnu khuru nnu"*, meaning *"400 meets 400"*. The translation for their expression for *"10 million"* is approximate – it meant something like *"there are so many things to count that their number is incomprehensible"*. These translations are not so strange in view of the etymology of large numerals in English. The word *"thousand"* for instance goes back to the Old-Nordic *"þushundrad"* and *"þus"* refers to the Indo-European root meaning *"to swell"*, *"to rise"*, or *"to grow"*. Thus, *"thousand"* roughly means *"a swollen hundred"* or *"a strong hundred"*.

2.4. Counting gestures

Claudia Zaslavsky stressed differences between pronunciation of numbers and corresponding gestures (Fig. 3; Zaslavsky, 1973). The Maasai north of the Tanzanian city of Arusha, seldom utter numbers without showing them with the fingers. For example, they bring the top of forefinger on the thumb and the top of the middle finger op the forefinger to indicate 3, and when the stretched forefinger rests on the stretched middle finger, it means 4.

In Rwanda and West-Tanzania, 4 is shown by holding the forefinger of one hand pushed against the ring finger, until it rests, with a snap, on the middle finger (Fig. 4). Eastern Bantu people say 6 = 3 + 3 and 8 = 4 + 4, while 7 becomes *"mufungate"*, or *"fold three fingers"*: 7 = 10 - 3. The Songora on their turn say 7 = 5 + 2, but, still, 9 is *"kenda"* or *"take away one"*: 9 = 10 – 1, *etc*.

2.5. Base 12

The origin of the twelve- and sexagesimal system, known from its actual use in the words "dozen" or "gross" and in subdivisions of time, is an often-returning issue for general mathematics journals (Ifrah, 1985). A commonly held opinion states the West owes it to the Babylonians, who divided a circle in 360°, but this only returns the question back in time to some 4000 years ago (and it could even be pushed back in time, to the Sumerians). There are two versions why the Babylonians on their turn used that base: the widespread *"arithmetic"*

Fig. 3 — A creativity of counting signs, from a 1910 "colonial source", document described as "Answer to 6134 of July 30, and to 6425 of August 20, 1909" and entitled "Report about native money, and the counting systems, in use in the district around 'Lake Leopold II'". Ethnographic File 126. Collection Royal Museum for Central Africa ©.

Fig. 4 — Sign for 6 in Rwanda (left) and gesture for 4 (a movement, shown on the middle and right figures).

and *"astronomical"* explanations showing the duodecimal (or sexagesimal) system presents an advantage if one knows the decimal method as well. The first emphasizes the simplicity in calculation when working with fractions, while the latter rests on the relation between the length of a lunar month and the approximate 12 months in a year. This underestimates the observational possibilities of the earliest people, and, after all, both are *a posteriori*. They do not provide a straightforward justification like for the finger base 10.

Yet, there is a related counting technique still in use in Egypt, in the Middle-East (Syria, Turkey, Iraq and Iran), Afghanistan, Pakistan and India (some include South-East-Asia as well). It uses the 12 phalanxes of one hand, counted by the thumb of that hand. The fingers of the left hand record the number of dozens, including the left thumb, since this one is not counting. As 5×12 = 60, it provides an indication why the numbers 12 and 60 often occur together (Fig. 5).

There is some "circumstantial evidence" too: matriarchal communities associate the number 1 with the woman, the number 3 with the man, and 4 with the union of man and woman. Alternatively, in a later evolution, 3 stood for man, 4 for woman. The number 4 was a wide spread mythical number, found in colors or social organization. The use of 6 as a mythical or sacred symbol was less shared than the 4-cult, but sometimes a mythology of a 4-cult changed into a 6-cult. For example, the four wind directions, North, South, East, West, were completed by two other points, zenith and nadir, when this was convenient. These simple examples were a motivation to use 3 and 4, when counting on the phalanxes of the hand.

In all, the reasons for using a base 12 remains a complicated question, an unsolved mystery even amplifying the mathematician's amazement, of which he can only wonder why it was used so extensively. Number words provide additional support for the extensive use of the duodecimal base. N. W. Thomas reported on the number words used by the Yasgua, Koro and Ham people in the region of the actual Nigeria (Thomas, 1920). They lived in an isolated region enclosed by rivers and maintained a particular vocabulary:

> *1 = unyi; 2 = mva; 3 = ntad; 4 = nna; 5 = nto; 6 = ndshi; 7 = tomva; 8 = tondad; 9 = tola; 10 = nko; 11 = umvi; 12 = nsog; 13 = nsoi (=12+1); 14 = nsoava (=12+2); 15 = nsoatad; 16 = nsoana…; 17 = nsoata ; 18 = nsodso; 19 = nsotomva; 20 = nsotondad …*

These Yasgua words have a Koro version:

> *1 = alo; 2 = abe; 3 = adse; 4 = anar; 5 = azu; 6 = avizi ; 7 = avitar; 8 = anu; 9 = ozakie; 10 = ozab; 11 = zoelo; 12 = agowizoe; 13 = plalo (=12+1); 14 = plabe (=12+2); 15 = pladsie; 16 = planar; 17 = planu; 18 = plaviz; 19 = plavita; 20 = plarnu …*

L. Bouquiaux gave a similar description, from the Birom region in central Nigeria (Bouquiaux, 1962):

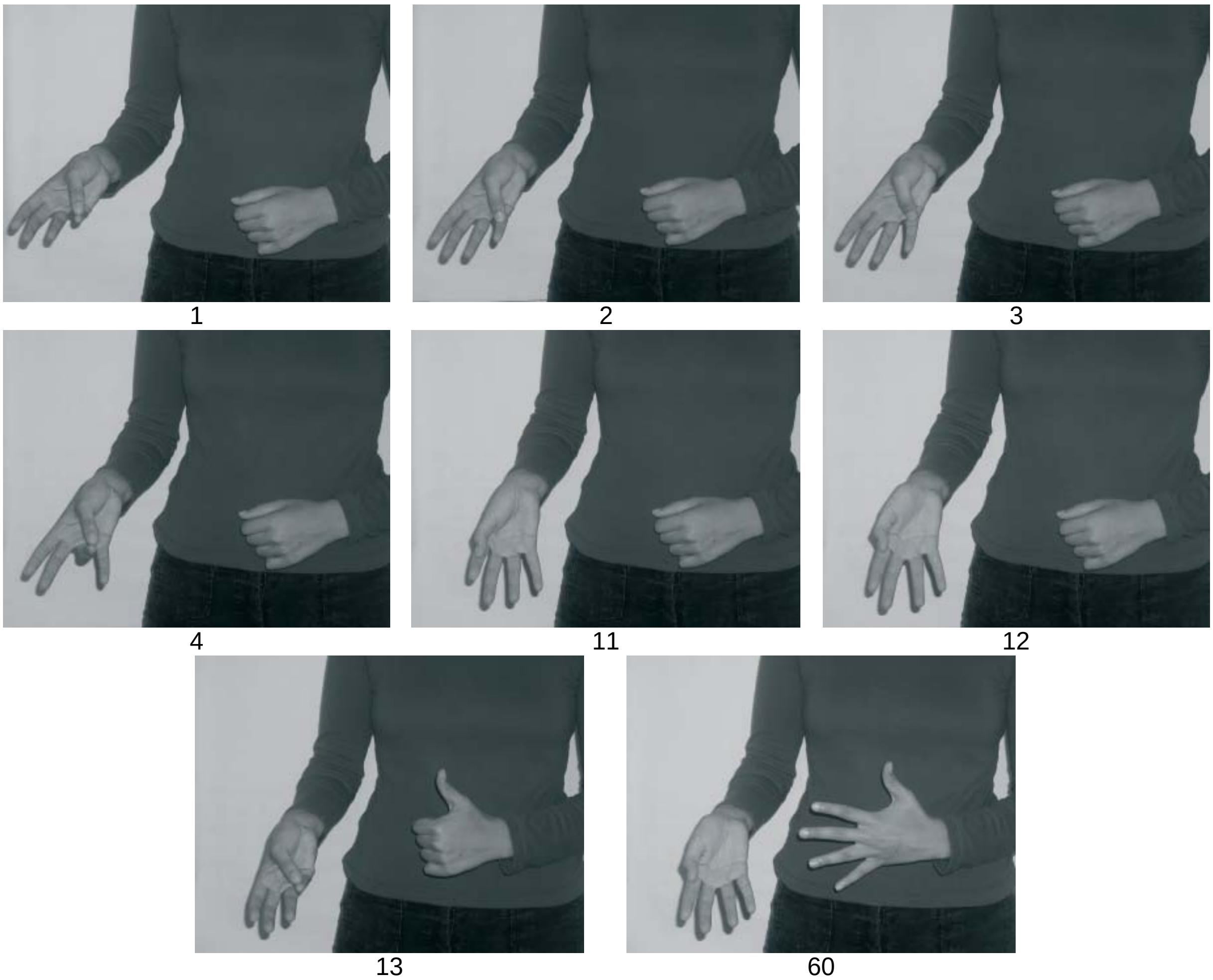


Fig. 5 — A duodecimal counting method.

1 = gwinì; 2 = bà; 3 = tàt; … 9 = aatàt (12- 3); 10 = aabà (12 – 2); 11 = aagwinì (12 – 1); 12 = kúrú; 13 = kúrú na gw gwinì (12 + 1); 14 = kúrú na v bà (12 + 2); 15 = kúrú na v tàt (12 + 3); … 20 = kúrú na v rwiit (12 + 8); … 24 = bákúrú bibá (12×2); … 36 = bákúrú bitát (12×3); … 108 = bákúrú aabitàt (12×9); … 132 = bákúrú aaggwinì (12×11); … 144 = nàga …

H. F. Mathews reported on still another community using a base 12 vocabulary (Mathews, 1917), and the Ishango region may hide another one...

2.6. Current linguistic studies

Didier Goyvaerts, professor at the VUB (Belgium) and at the National University of Congo in Bukavu, co-supervised a research thesis on African number words by Sofie Ponsaerts (KU Leuven, Belgium; Ponsaerts, 2002). He provided her with his results on the Logo language, spoken in the North East of Congo. The approach followed the example of the British language expert James Hurford and is itself rather abstract, almost of mathematical nature. Here, mathematics again illustrates its universal strength as it allows the formulation of linguistic rules valid for all languages despite their enormous diversity:

1 = alo; 2 = iri; 3 = na; 4 = su; 5 = nzi; 6 = kazya; 7 = nzi drya iri (= 5 + 2) ; 8 = nzi drya na (= 5 + 3); 9 = nzi drya su (= 5 + 4);
10 = mudri; 11 = mudri drya alo; 12 = mudri drya iri; 16 = mudri drya kazya (= 10 + 6); 17 = mudri drya nzi drya iri (= 10 + 5 + 2); 18 = mudri drya nzi drya na (= 10 + 5 + 3); 19 = mudri drya nzi drya su (= 10 + 5 + 4);

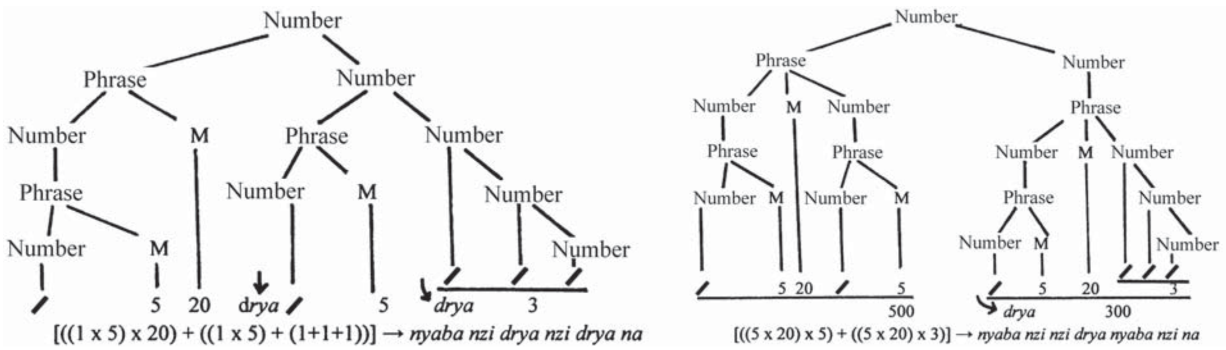


Fig. 6 — Diagrams from Ponsaerts' dissertation, for forming 108 (left) and 800 (right; see Ponsaerts, 2002).

20 = nyaba alo (= 20 × 1); 30 = nyaba alo drya mudri (= 20 × 1 + 10); 40 = nyaba iri (= 20 × 2); 50 = nyaba iri drya mudri (= 20 × 2 + 10); 60 = nyaba alo (= 20 × 3); 70 = nyaba na drya mudri (= 20 × 3 + 10); … ; 100 = nyaba nzi (= 20 × 5);
200 = nyaba nzi iri (= (20 × 5) × 2); 300 = nyaba nzi na (= (20 × 5) × 3); … 600 = nyaba nzi kazya (= (20 × 5) × 6); 700 = nyaba nzi nzi drya nayba nzi iri (= (20 × 5) × 5 + (20 × 5) × 2); 800 = nyaba nzi nzi drya nayba nzi na (= (20 × 5) × 5 + (20 × 5) × 3); 700 = nyaba nzi nzi drya nayba nzi su (= (20 × 5) × 5 + (20 × 5) × 4);
1000 = nyaba nzi mudri (= (20 × 5) × 10).

Ponsaerts accompanied her illustrations by following comment:

The symbol / is the semantic representation for the word "one" or a "unity". For instance, the group of symbols ////////// corresponds in the same way with the word ten, but also with the related morphemes "-ty" and "-teen". […] M can in the case of the English language for example be "-ty", "hundred", "thousand" etcetera.

Funny enough, this linguistic research uses the slash "/" as a notation for the unit number (Fig. 6), similar to what may have done the earliest civilizations at the sources of the Nile.

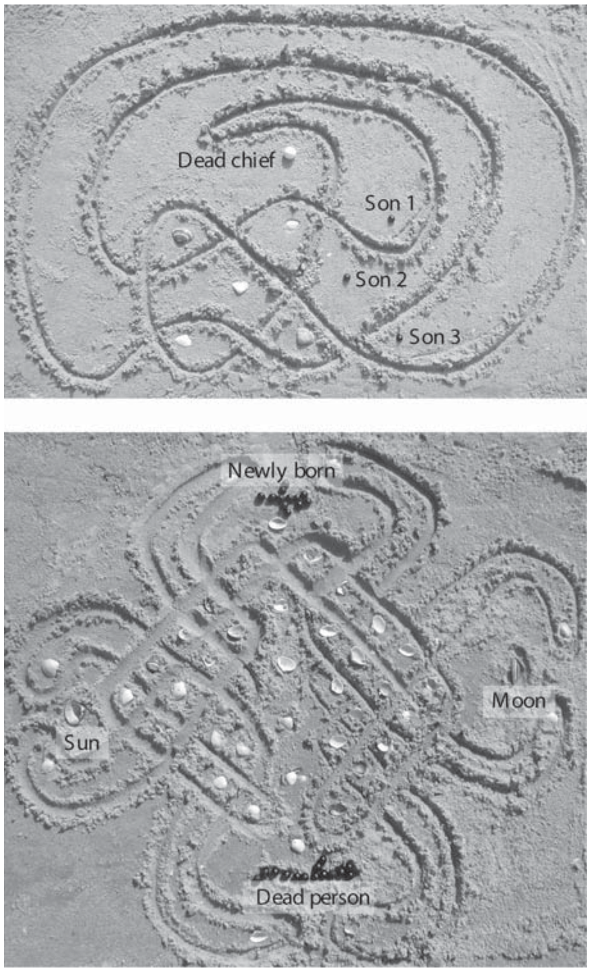


Fig. 7 — A graph explaining who becomes the new chief (above), and the "graph of life" (below; see Ascher, 1988).

3. DRAWINGS (GEOMETRY)

3.1. Graphs, mathematics and Africa

Graph theory forms a standard part of mathematics, ever since these "networks of (oriented) arrows" could be "calculated" using matrices, allowing conclusions that would otherwise have been unpredictable in a tangle of lines. Different cultures around the world have a custom of making drawings in the sand, with a finger or a twig to reinforce the mythical character. It shows not only dance, music and dramatic expression were sources of inspiration for rituals, but mathematical schemes as well.

The Tsjokwe, for instance, from the border area of Congo and Angola made *"sona"* diagrams (Fig. 7; previous page). During the initiation ceremony, each generation learnt how to execute them, and sometimes these drawings were used in mourning. The story goes that when a village chief died, three candidates tried to catch the heritage. A geometrical drawing represented this situation: a large white dot in the middle for the dead chief, and three small black dots (numbered by 1, 2 and 3 in the illustration). A closed curve surrounded the three applicants and the dead chief. Two beleaguers could not reach the dead chief without crossing the line (1 and 2), but 3 could, and thus he became the new chief. Another example is the graph of "life", where a single curve surrounds a dead person and a newly born.

A technique for memorizing the drawings weaves a curve around some given points, while the inner edges of the rectangle are imagined to be mirrors, reflecting the curve like a ray of light. As it crosses the network, a square can be colored black, white, again black, and so on, until the entire network is colored. It creates different patterns, depending on the points and mirrors (Fig. 8). The reverse operation is sometimes executed in graph theory for chess moves on a checkerboard.

Marcia Ascher made several lists of drawings classifying them with respect to region or people, and emphasizing properties of mathematical nature such as the doubling a graph

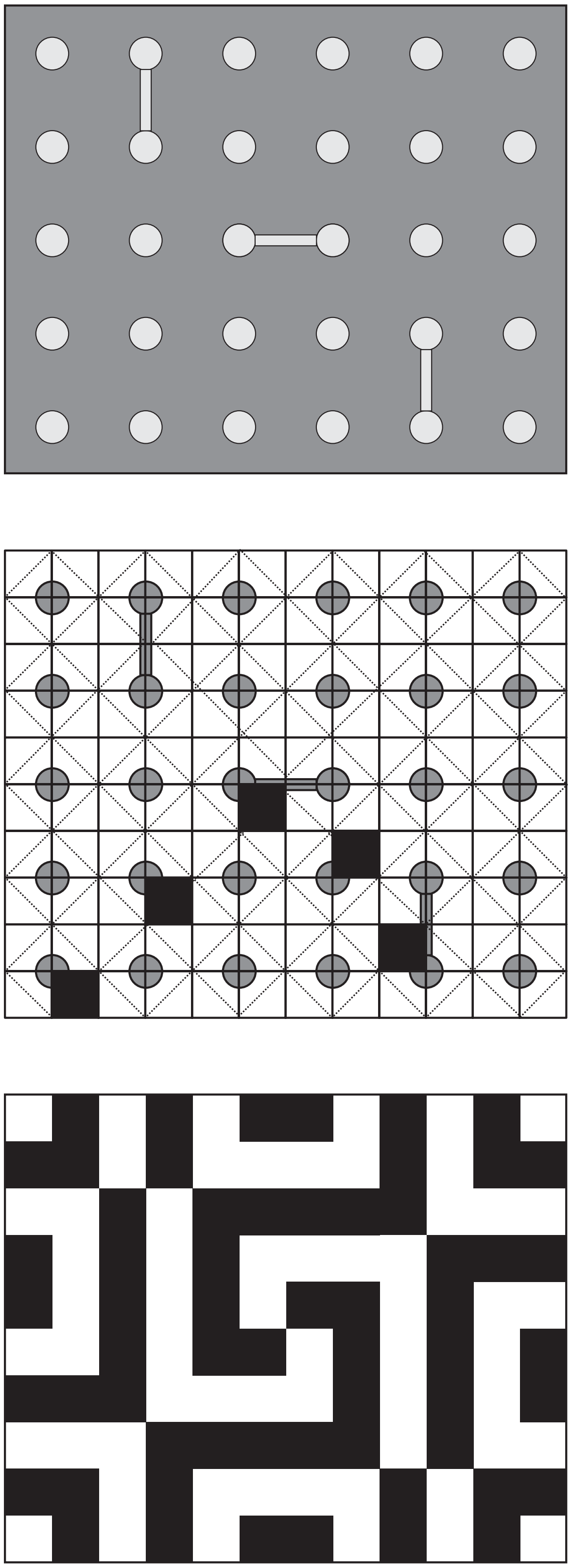


Fig. 8 — Five rows of six dots and two "mirrors" (above), where an imaginary ray of light successively colors squares (middle), so that the final pattern (below), which brings African fabrics to mind, rather than curves (Gerdes, 2002).

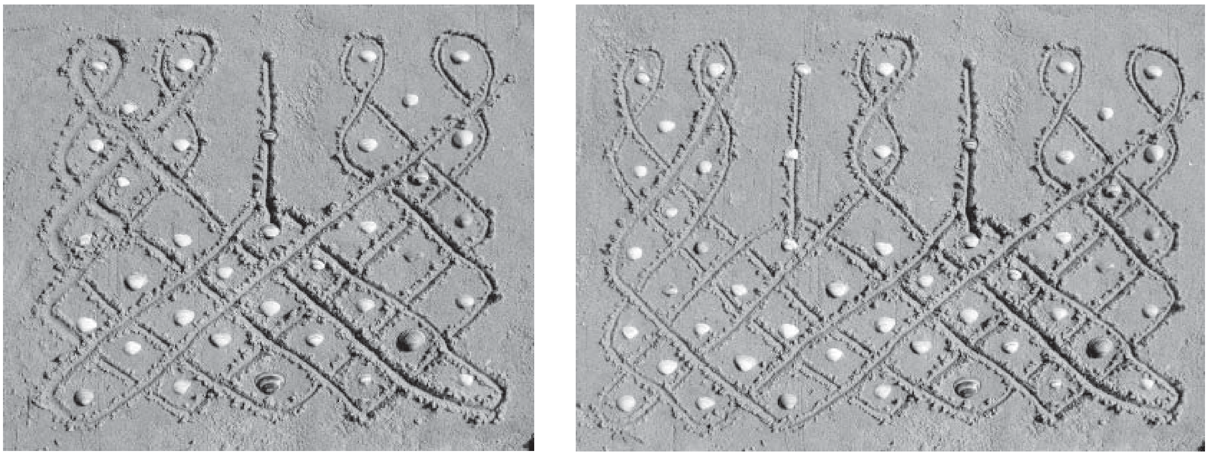


Fig. 9 — When the dots represent Myubo huts, and their number almost «doubles», so does the corresponding graph (see Ascher, 1988).

(Fig. 9), or its symmetries (Ascher, 1988; 1994). P. Gerdes formulated some truly mathematical theorems about these African curves, by associating matrices to them and introducing operations on the corresponding numbers (Gerdes, 1999). After all, it seems philosopher L. Wittgenstein was right, when he tried to define the essence of mathematics using these graphs as a typical example: *"everyone would immediately recognize its mathematical character"*.

3.2. Geometric patterns

African woven fabrics sometimes have a mathematical inspiration as well. The Ghana Ashanti drawings compete in renown with South-African patterns, but Central-Africa shows many geometric drawings too, on enclosures around the huts, baskets, milk jars, or ornamented drums (Fig. 10; Pauwels, 1952).

Mathematician Celis wondered why non-figurative drawings enjoyed a strong preference above images of people, animals or illustrations telling stories (Celis, 1972). He studied hut decorations in an isolated part in the Southeast of the Rwanda, and found out the so-called *"imigongo"* went back in time for about three hundred years, to the legendary king Kakira ka Kimenyi. They can be reduced to a mathematical algorithm using only parallel lines and some given skew directions, and rhombi, isosceles and equilateral triangles. His geometric observations led to the rejection of some drawings as unoriginal, while others used the Kakira rules on computer to program *"traditionally correct"* patterns for new fabrics.

Another mathematical way to appreciate African geometrical figures is their classification using so-called "group theory", inspired by crystallography. Donald Crowe studied these repetitive patterns in the art of the Bakuba of Congo and in Benin (Crowe, 1975), and found all 7 mathematically distinguished frieze patterns (Fig. 11).

3.3. Further geometric studies

The abundance of geometric examples is immediately evident during a visit of the Africa Museum of Tervuren (Belgium) and it explains the fertility of ethnogeometry (Fig. 12). Crowe for instance also studied the 17 mathematical possibilities of two-dimensional plane patterns, and in Benin he found only 12 cases. In another collection of ornaments he used involved algebra to distinguish sub-classifications in given art collections. Frieze patterns with one additional color, again allow other mathematical excursions, as the number of patterns increases from 7 to 24.

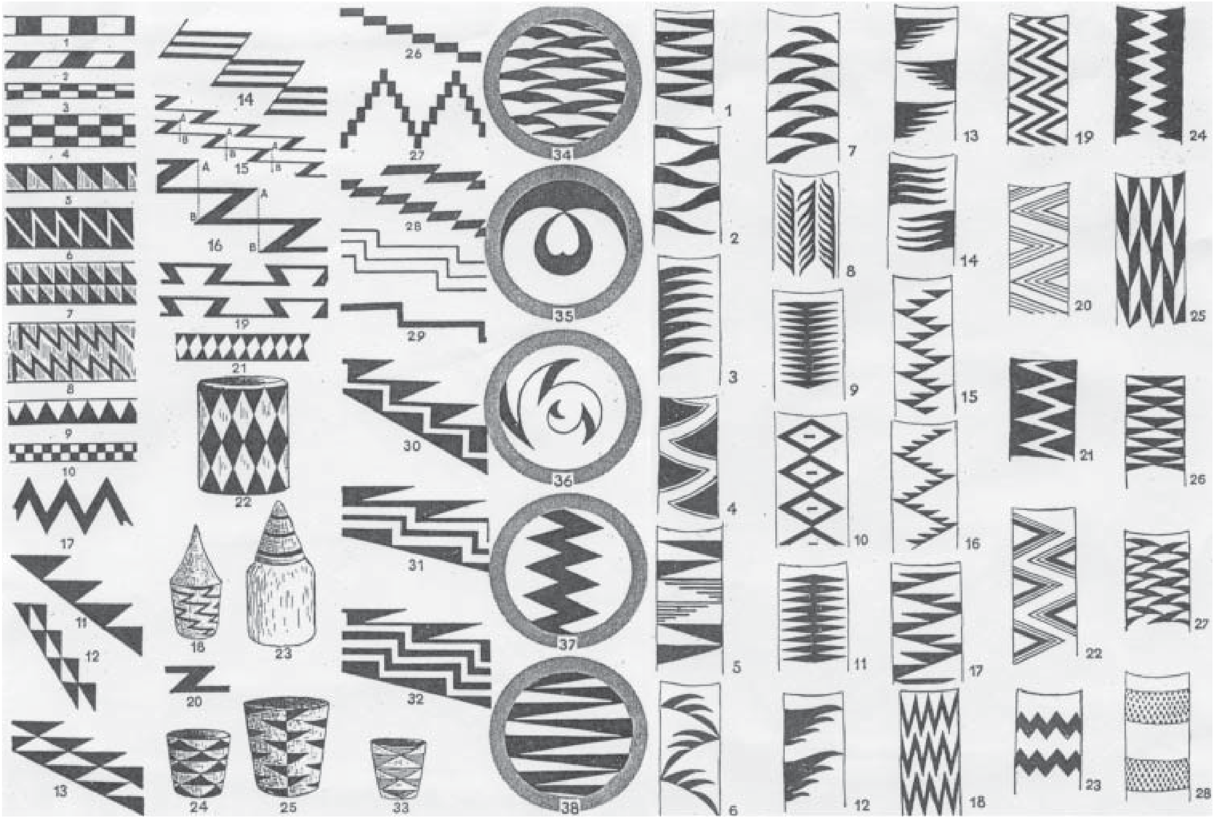


Fig. 10 — A list of Rwanda drawings, the number indicates an associated description from author Pauwels (1952: 479-480).

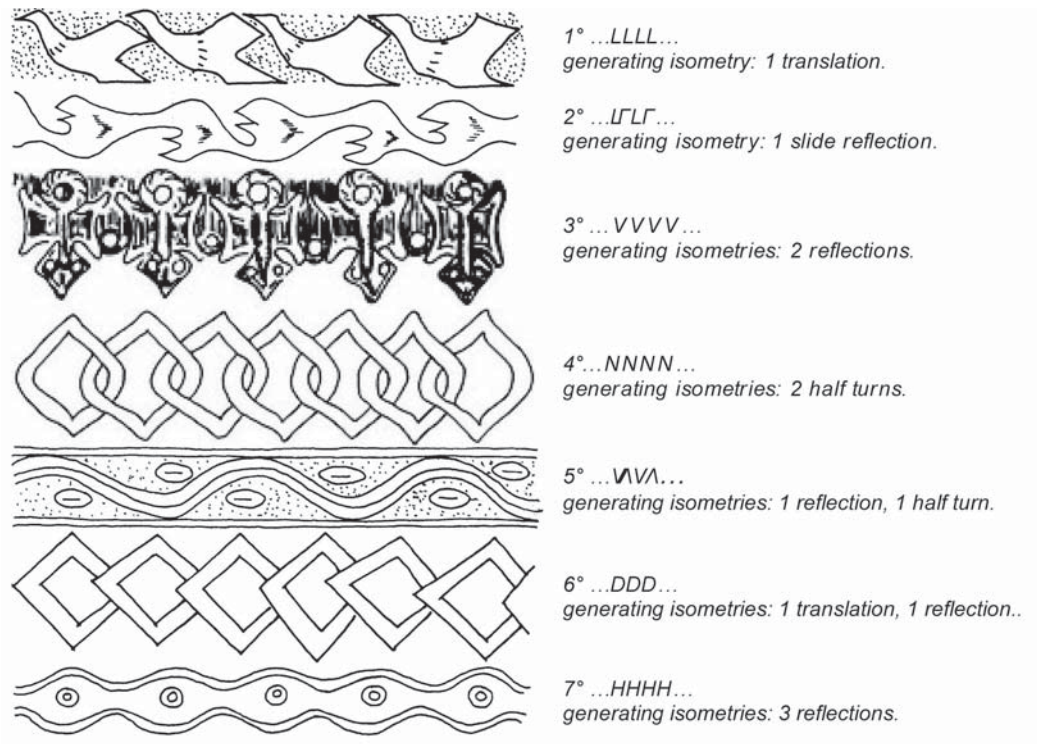


Fig. 11 — In Benin, all 7 mathematically possible frieze patterns can be found (Crowe, 1975).

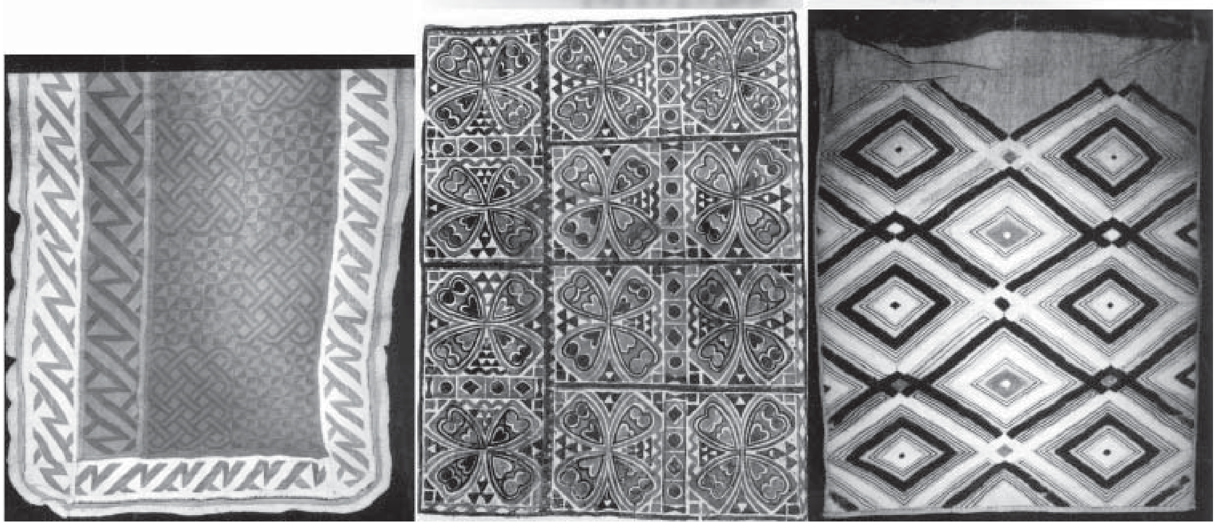


Fig. 12 — Geometric patterns, on a plate, a hat, a screen (first row) or on woven fabrics. In the given order: EO.0.0.33710; EO.1949.21.1; EO.0.0.41670; EO.1980.31.2; EO.1968.39.3; EO.0.0.25651.
All objects: collection Royal Museum for Central Africa ©.

The theory of fractals for the study of African art appeals to yet different mathematical machinery, of even more recent times, and it even filled a complete book, on *"African fractals"*.

Thus, a discovery of a mathematical pattern on, say, a carved bone from Central Africa, is at present not as strange as it was in colonial times.

4. Dawn of number notation

4.1. Lines in the sand

Drawing in the sand immediately leads to the beginning of number notation. The Bashongo from East-Congo would draw lines in the sand with three fingers of one hand, and after three groups of three, complete the ten by one line: ||| ||| ||| |, but architect Tijl Beyl (Belgium) revealed the work of Dr. Mubumbila Mfika, a Gabon chemist, who stressed a longer line at the end of each triplet revealed the Bashongo counted per three, and the longer line would simply be a symbol for a counting interspacing (Mubumbila, 1988; 1992). The Bambala would then, again like in the West, count per five, and place a interspacing between the groups. In the counting method of the Bangongo, Bohindu and Sungu, a grouping by four could be distinguished (Fig. 13).

The gestures reflect the creativity in counting words, or vice versa, and even notational aspects, using three or four fingers in the sand, can have influenced a preference for one base or another, explaining once again the differences in number bases in Africa. Zaslavsky pointed out counting gestures sometimes transform into a notational system: the Fulani of Niger and Northern Nigeria, put two short sticks in the ground in the form of a V, to indicate they possess 100 animals, while a cross,

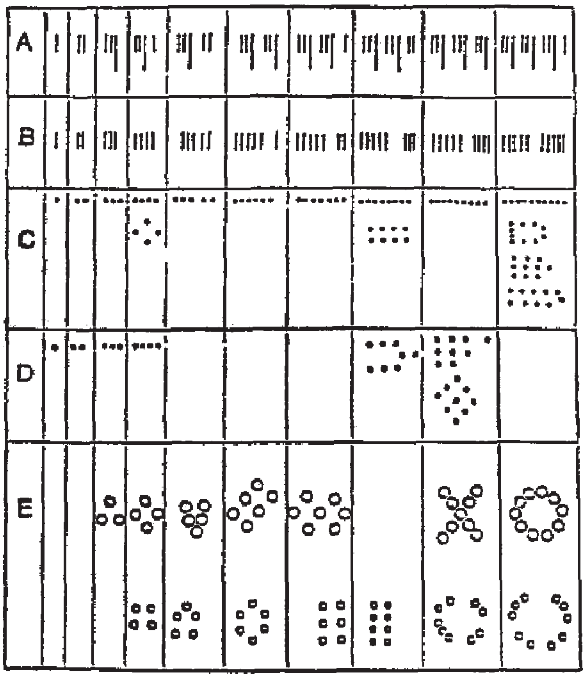


Fig. 13 — Table of Mubumbila Mfika, with graphical counting symbols and their grouping, by different Bantu people: A = Bashongo; B = Bamabala; C = Bangongo; D = Bohindu; E = Sungu (see Mubumbila, 1988).

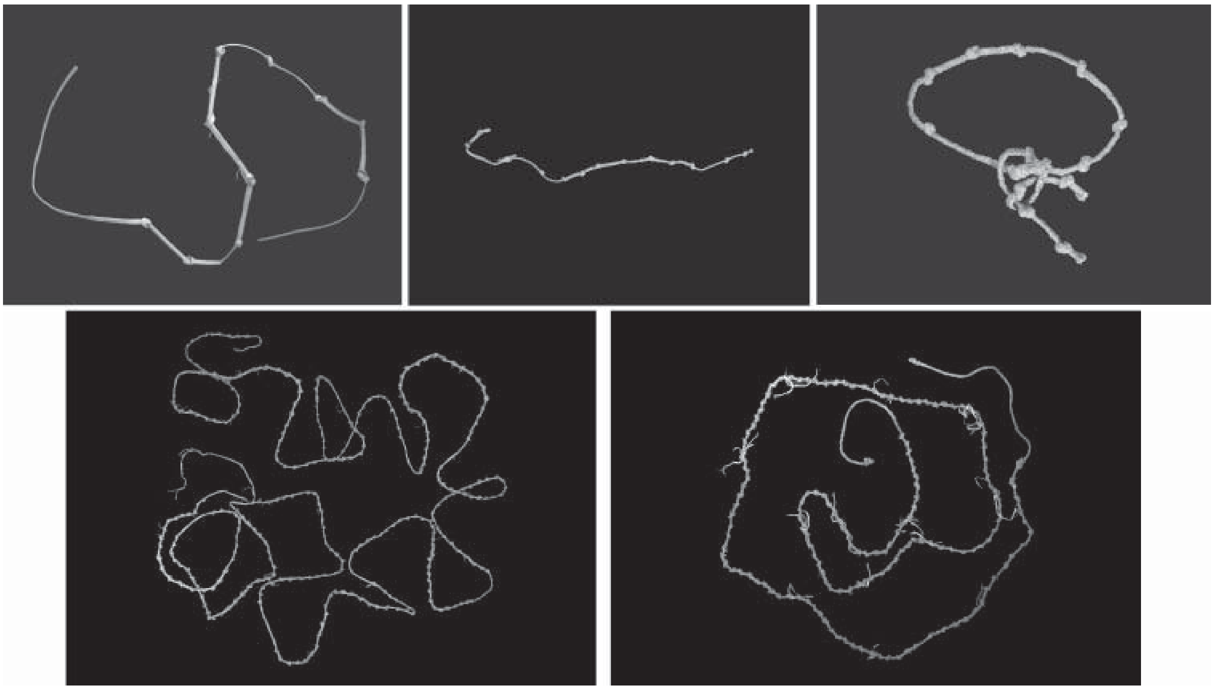


Fig. 14 — Congolese counting strings: Row 1: 10 knots (left), 12 knots (middle and right), objects found in Kabinda, DR Congo. In the given order: EO.0.0.1960-4; EO.1949.63.22. Row 2: String with 10 times 30 knots, representing the number of gestation days for a cow (left), object found in Luviri, close to Nyundo, Kibiribiri people, DR Congo; and a money string, where each knot represents one “mosolo”, a kind of local money used for dowry, object found in the ‘Basoko’region. In the given order: EO.0.0.1816; EO.0.0.884.
All photographs: ADIA, Brussels – Belgium. All objects: collection Royal Museum for Central Africa ©.

X, indicates 50. Horizontal sticks indicate tens, vertical sticks units, and thus, = | | | means 23, while VVVVVVX| | stands for 652.

4.2. Knotty counting

It is but a small step to the different supports used to easy counting. The common knotted strings make any ethnomathematician immediately think about Inca quipus, but their use was different in Africa (Fig. 14). They often were simple memory devices for recording data about the passing of the days, weeks, months or years, or for registering payments, loans, distributions of water, goods to transport, sold ivory teeth, *etcetera*.

Knotted ropes were spotted in the expedition of Grant and Speke, where carriers used them, that is, long before missionaries could have imported them. Knots were tied on both sides of a string, at one side for the sale, at the other for purchase. A debtor gave a string to his creditor in guise of promise, and if the creditor lost the string, he lost his right for repayment too. In the kingdom of Monomotopa the court historian was bound to tie one knot at the accession to the throne of a new king. In 1929, this rope had 35 knots, and all kings could be identified, going back to the middle of the fifteenth century.

S. Lagercrantz dedicated three lengthy survey papers to African counting strings, counting sticks and cuts or tattoos on the body, and he made maps about their dispersion (Fig. 15; Lagercrantz, 1968; 1973).

4.3. Sticky numbers

The custom of using sticks in wood, bone, or stone, to denote numbers, was widely spread too. Figure 16 shows examples from the treasures of the Africa Museum of Tervuren (Belgium), while figure 17 provides a map and two special examples. In the beginning of the seventies, archaeologist Peter Beaumont discovered a remarkable fibula of a baboon during excavations in caves in the Lebombo Mountains, on the border between South Africa

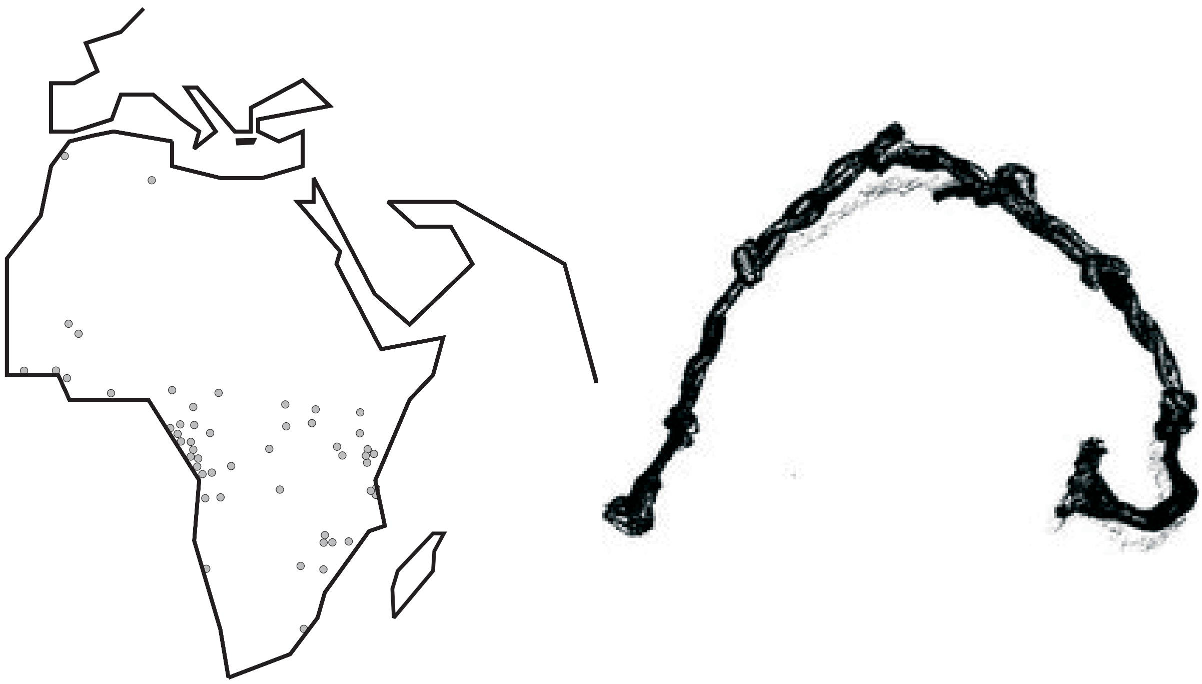


Fig. 15 — Lagercrantz' map of counting strings (left), and a counting string made after one of his examples (right). Drawing by Arch. Frederic Delannoy, Sint-Lucas Brussels.

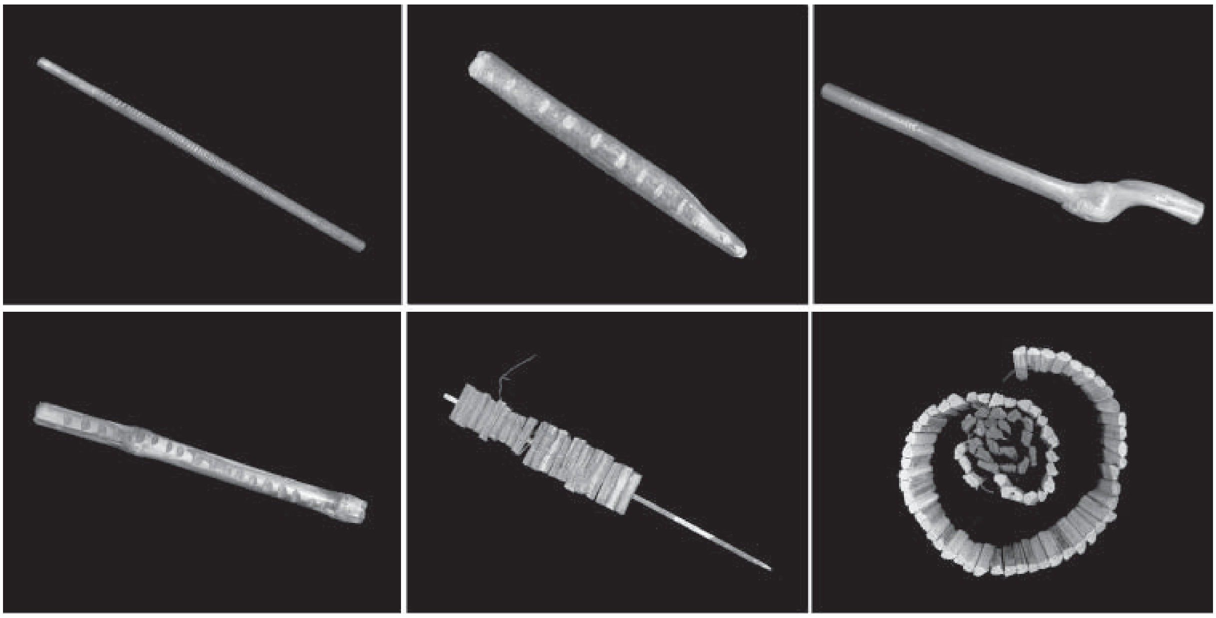


Fig. 16 — All kinds of counting sticks with clear carvings, and counting devices using little rods held together by a stick or a rope. In the given order: EO.0.0.1538 (origin: Ababua people, Ubangala region, North of Kisangani, DR Congo); EO.0.0.1816; EO.0.0.2144; EO.0.0.34992; EO.0.0.2111-1; EO.0.0.9836. All photographs: ADIA, Brussels – Belgium. All objects: collection Royal Museum for Central Africa ©.

and Swaziland (Bogoshi, 1987). The 7.7 cm long bone appeared to be 35,000 years old and showed 29 clearly delimited carvings.

A European example is a 17 cm large wolf bone engraved in 1937 in Vestonice by Karl Absolon in the former Czechoslovakia on which about 30,000 years ago 57 thin lines

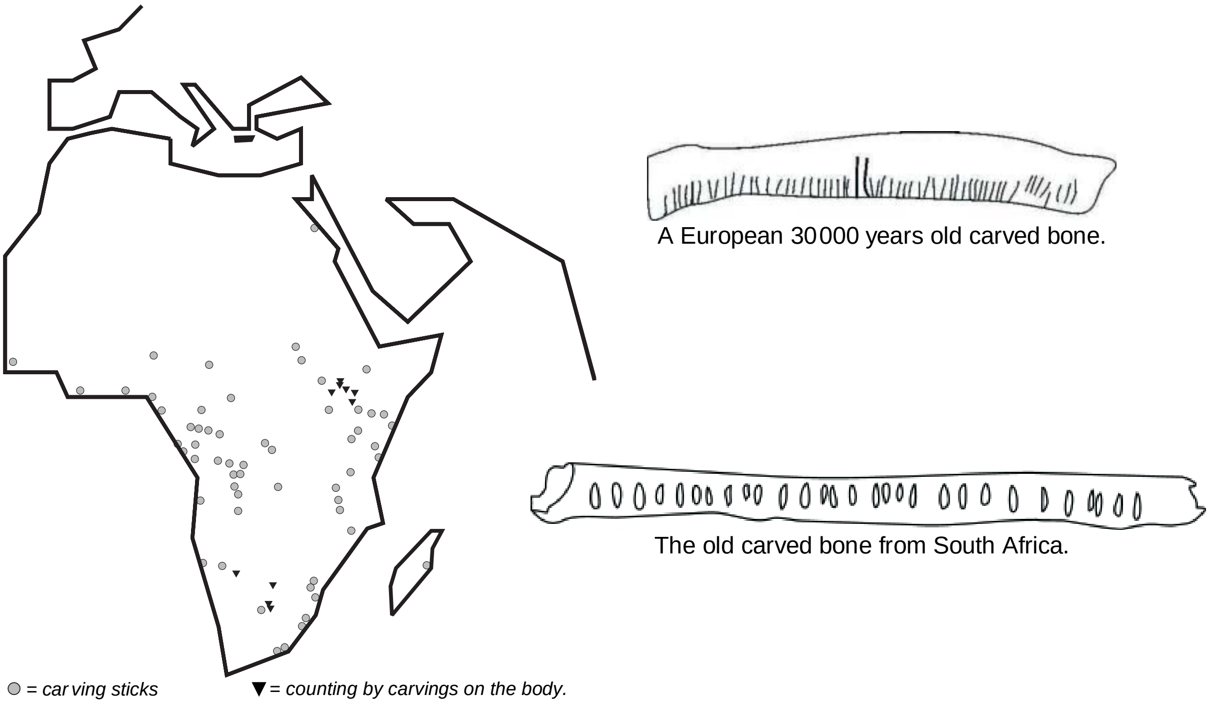


Fig. 17 — Lagercrantz' map about the use of counting sticks and carvings on the body (left) and two special examples (right). Drawings by Arch. Frederic Delannoy, Sint-Lucas Brussels.

were carved. The first 25 are presumably drawn in little groups of five and all have the same length. They are marked on the end by a carving twice as long, after which again a long carve announces a group of 30 thin lines. The finding may show a counting method referring to the five fingers of the hand, but very little is known about the people who carved this bone, except they also left a small ivory statue, both now in the "Moravske Muzeum".

On the map, counting with tattoos was indicated by ▼-symbols, at the North East of Lake Rutanzige and in the Southern African region of the Kalahari Hottentots, an often-recurring link to the Ishango region:

The practice to enumerate the number of killed enemies by carvings on the own skin of the warrior has a specific and well-known distribution, but is seen as archaic. On one side, there is the region around the Lake Albert (Rutanzige), while on the other side there is the central part of South Africa. This reflects the well-documented ethnographical relations between North East and South Africa.

Lagercrantz also wondered why these interesting counting objects are so rarely shown to the public in Africa museums, while the practice seemed to have been so abundant:

Counting by means of stones, nuts, sticks, blades of grass and similar objects is an old practice that is in particular remarkable for South-Sudan. The counting method can be compared to counting strings. The latter are spread more to the North, but they are part of a same black culture. The ancient cultural elements also include the counting by lines drawn on the ground or painted on doors and walls.

Maybe the actual computer age makes strings, sticks, stones or bones, seem too unusual to be number records, and this could explain the lack of attention Lagercrantz was complaining about. However, even in many non-African cultures these number "notations" were popular. The Code Napoleon referred to "la taille" or "the tally", a carved stick on which received taxes were recorded, while the modern Arab "subha", or the Jewish "tallit" both have knots to count preys. Japanese sell strings with 30 little pearls in their Shinto temples, and many Catholics still count prayers on the 60 knots of a paternoster. Thinking of the 12 Apostles, this 12-60 biblical link suggests missionaries may have imported to Africa what originally came from the continent.

4.4. African pre-writing

In a broader context, there are proofs of the existence of memory aids to keep track of quantities, or data in general, or even complete stories and messages. Some older civilizations (Nubia, Kush), and in more recent times the Vai in Liberia, developed notational systems, and certain pre-hieroglyphic forms of were found. The Africa Museum of Tervuren (Belgium) shows cushions with images of the tasks to be remembered, stylized drawings representing a story, as if it were a pre-hieroglyph, and ropes with of objects thread on it (Fig. 18). Some objects correspond to proverbs to keep in mind, such as "You who love dancing, never dance on the top of a spear" or "a child is a prawn: when you carve it, it will let you cross (the river)".

5. Reasoning without writing

5.1. Mental arithmetic

Writing and mathematical reasoning seem inseparable. *Even some animals can recognize quantities and so knowledge of numbers is hardly sufficient to talk about* "mathematics", some die-hards will assess. Of course, it is more difficult to reconstruct proofs of "abstract" thinking, by definition of the word abstract. There are legends of people less interested in a missionary's tables of Moses than in the tables of multiplication he brought along. There are a few written sources too: A report from 1788 was revealed by Fauvel and Gerdes, about the *"wonderful talent for arithmetical calculation"* of an African slave, Thomas Fuller (Fauvel, 1990). He was brought to America when he was 14 years old and had developed his mental arithmetic skills when still in Africa. In the report, two *"respectable citizens of Philadelphia"* related how this native of Africa, who could neither read nor write,

Fig. 18 — Cushions, storyboards and story ropes. In the given order: EO.0.0.42871; EO.0.0.27312; EO.1975.48.1. All objects: collection Royal Museum for Central Africa ©.

could, for instance, compute in two minutes how many seconds a year and a half counts, or how many seconds there are in seventy years, seventeen days and twelve hours old (including lap years).

5.2. African mind games

John Von Neumann is often seen as one of the founders of mathematical game theory. Today, exotic games are still seen as challenges for testing artificial intelligence on computer, since games not yet fully described imply a computer program cannot simply consist out of lists of all past winning games nor simply rely on the greater calculating power of a machine. The non-European game "Go" drew the mathematicians' attention for some time, while in Africa it were the games of the mancala type (the Dogon "sey" or the East-African "yoté" created less interest – Fig. 19).

The African strategy games of the mancala family are among the oldest mind games

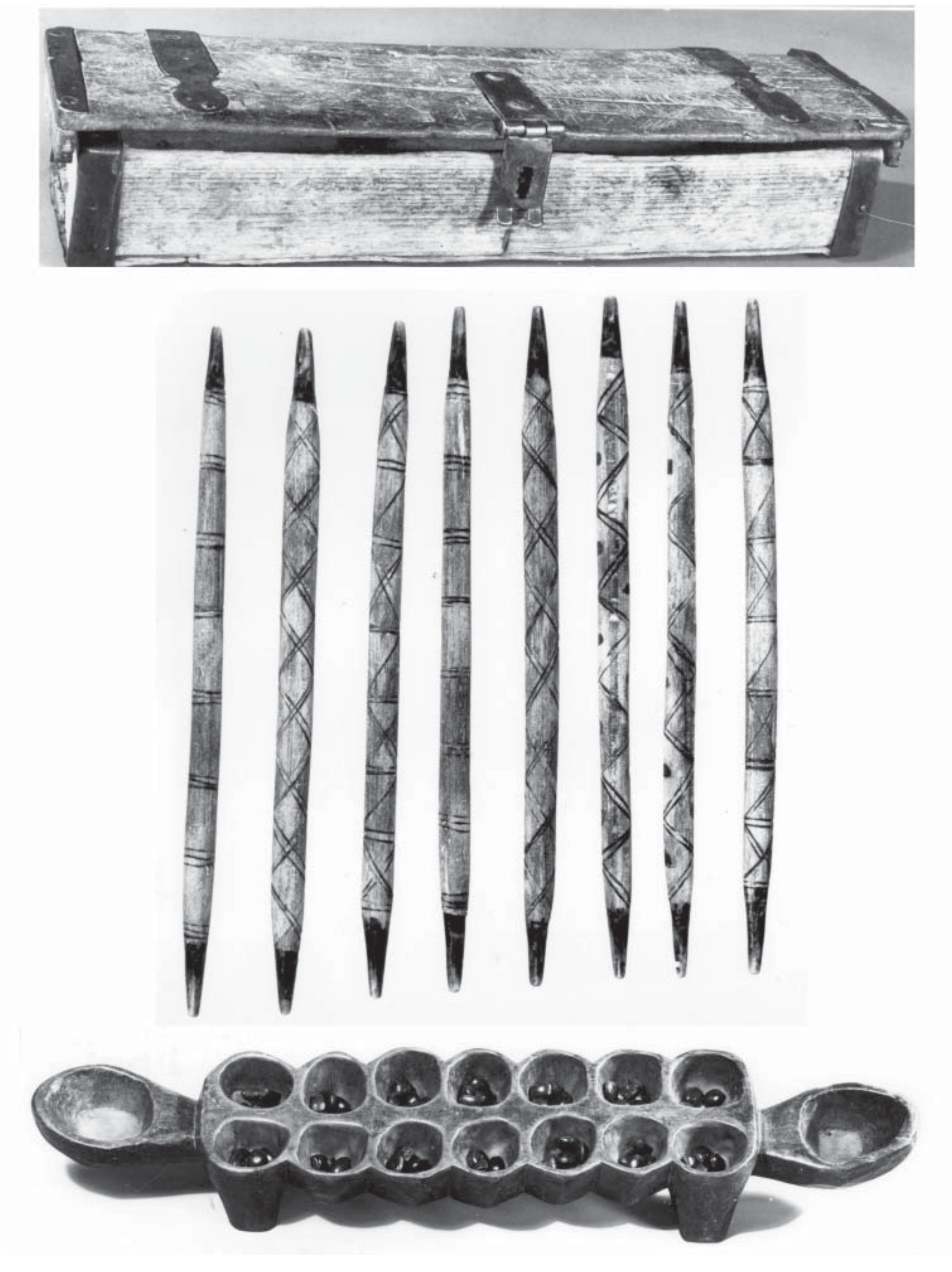


Fig. 19 — A "heads or tails" game and a remarkable seven holes mancala type game. In the given order: EO.1974.54.37-1; EO.1974.54.37-2 to EO.1974.54.37-9; EO.1969.59.636. All objects: collection Royal Museum for Central Africa ©.

of the world, and often classified among of the world's top games (Bell, 1988). There are different regional names such as boa, okweso, ayo, (igi)-soro, etc., and versions, with two to four rows and 6 to 50 holes in a row. The rules of the game vary and in this way it is possible to study cultures, not with respect to their language or dialect, nor to their traditions or religion, or to their music of artistic expressions, but to their way of thinking (Fig. 20).

5.3. An example: a simple mancala game

We outline a short version of a game, to illustrate reasoning involved in an African game and to prepare the reader for a difficult multiplication explained below. Trying out the game in the sand may be more useful than reading the present text, in order to appreciate its characteristics! Well, each player disposes of 6 holes, those directly in front of him, which compose his own camp, and one so-called "kalaha", often a larger hole, situated at the right of his own camp. The goal of the game is to capture as many pawns as possible from the opponent. In the beginning of the game, there are two pawns in each hole (or more, for advanced players), except in the kalahas at the edge. The first player starts by picking up all pawns in one of the holes, at his choice, and spreading them around the board, counterclockwise, one by one, one in each hole. A pawn is dropped in the own kalaha too and in the holes of the opponent, if there are enough pawns. If the last pawn of a player lands in the own kalaha, he gets a second turn.

"Making a bridge" is an often admitted additional move. It means that if the last pawn falls in a hole with other pawns already in it, he can take them and continue with these extra pawns. This may also happen in the camp of the opponent, with pawns of the opponent. Capturing pawns from the camp of the opponent happens when the last strewn pawn lands in an empty hole of his own camp. The player takes all pawns from the hole of the camp of the opponent and puts them in his own kalaha together with his pawn.

When all twelve holes in the middle of the board are empty, the game ends. The winner is the player with the most pawns in his kalaha.

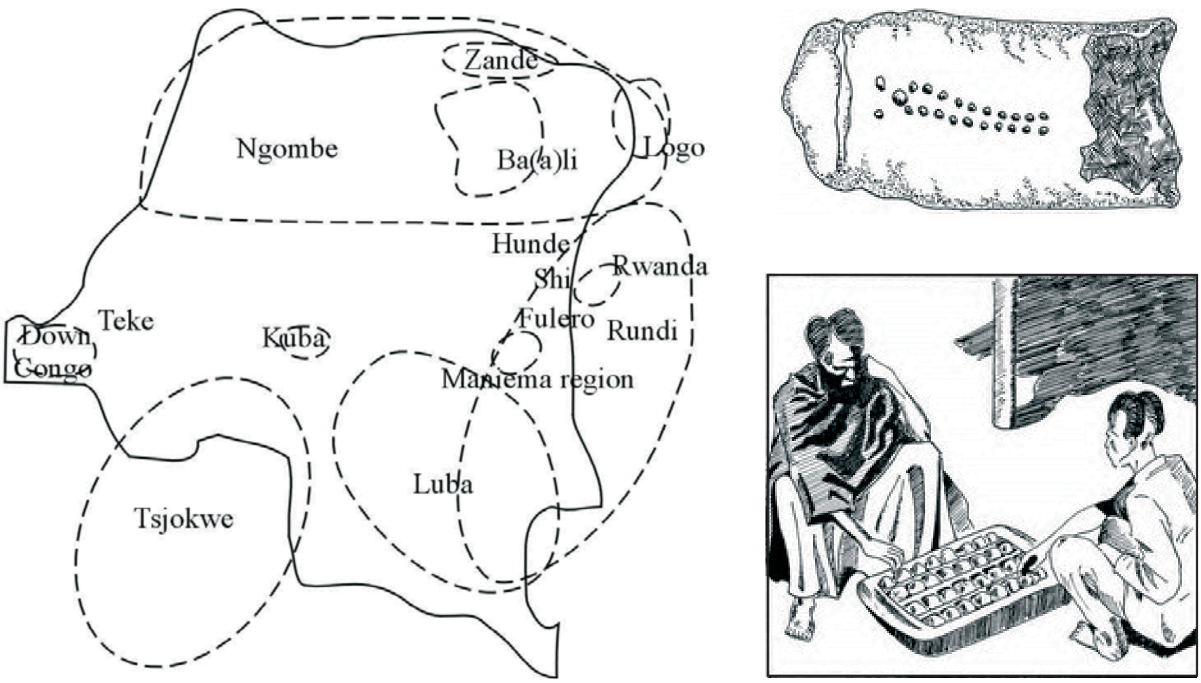


Fig. 20 — A map of Congo with regions where the mind game is played similarly (left), an old stone board (above right), and a modern mancala board with four prepared pawns. Drawings by Arch. Frederic Delannoy, Sint-Lucas Brussels.

When the game is almost over, it may happen that one of the players has no more pawns at his disposal to play. He can skip his turn and wait as another pawn from the camp of the opponent drop in his own camp.

Here are the first moves of a game, with only 2 pawns (shells) and 5 holes. To ease notations, we indicate the number of shells in the camp of a player with five digits, followed by the number of already obtained shells in his kalaha. Thus, the initial situation is (2,2,2,2,2; 0), for both players. Now suppose the first player moves two shells from the fourth hole: (2,2,2,0,3; 1), so that he gets an additional turn, because the last shell was dropped in his own kalaha. He decides to empty the second hole: (2,0,3,1,3; 1). The last shell falls in an empty hole of his camp, so that two shells of the opponent can be obtained. Together with the own shell, they are dropped in the kalaha: (2,0,3,0,3; 4), while the opponent still has (2,0,2,2,2; 0): the score is 4-0.

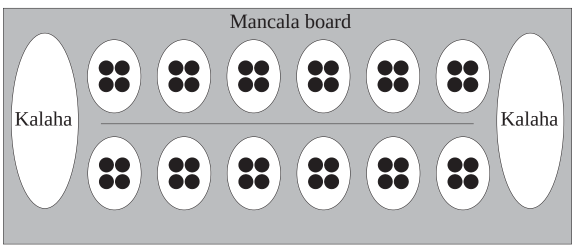
Now it is the opponent's turn: he decides, for example, to spread the first hole (0,1,3,2,2; 0), and, making a bridge (0,1,0,3,3; 1). He has a second turn, and empties the second hole: (0,0,1,3,3; 1). The shell fell in empty hole, in his camp, and thus he is allowed to put the 3 shells of the first player and his, in the kalaha: (0,0,0,3,3; 5), while the first player looks at a (2,0,0,0,3; 4) situation. The score is now 4-5: the tide has turned, for the second player, but the game is far from being finished. Note the overwhelmed reader can practice further, playing the related Bantumi game on a cell phone of the Nokia 3310 series.

5.4. Research mathematics about African games

Duane M. Broline and Daniel E. Loeb published about the combinatorics of two mancala type games, "ayo" and "tchoukaillon", a Russian equivalent (Fig. 21; Broline, 1995). The solitary version is played in different holes with one additional larger hole, the cala, and the aim is to get all pawns in the cala, using rules similar to the already explained mancala game.

At first, the pawns are sowed in an arbitrary hole. Next, they are put one by one in the successive holes in the direction of the cala. If there are too many, the player continues from the hole opposite to the cala. Again, if the last pawn falls in the cala, the player gets an additional turn, but if it falls in an empty hole, the game stops. If the last pawn falls in a hole with pawns in it, the player sows these pawns further one, all together, making a "bridge".

Numbering the holes starting from the cala, Broline and Loeb could show that if s(n) is the smallest number of pawns necessary for the nth hole to lead to a game the player can win (if he plays cleverly), the number of pawns leading to a winning position approaches n^2/π, for large values of n (asymptotically, mathematicians would say). Their computations involve high-level analysis, including the so-called gamma function Γ and the hypergeometric function F. The result is impressive, because the mathematical constant $\pi = 3.14159265\ldots$ pops up in the context of an African game. Furthermore, awari, another variant of mancala, could be entirely "solved" by Dutch



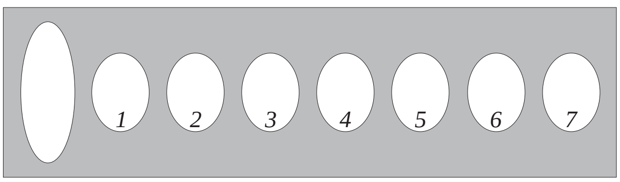


Fig. 21 — A mancala game board with the pawns in initial position (left) and the similar tchoukaillon board (right).

researchers John W. Romein and Henri E. Bal (Romein, 2002). They used a parallel computer with 144 Pentium III processors of 1 Ghz, 1 petabyte of information, a disk space of 1.4 terabyte, and 51 hours computer time. They proved the game is a perfect mind game: there is no advantage or disadvantage to open the game (Irving, 2000).

6. COMPLEX REASONING WITHOUT WRITING

6.1. Yoruba calculations

The Yoruba live in the South-West of Nigeria and in parts of Benin. In 1886 already, Dr. O. Lenz noticed people in the region of Timbuktu were involved in mysterious calculations he could hardly understand (Nicolas, 1979). Mann would give a more comprehensible report in 1887, describing the activities of "the magician-calculator". Yoruba number names refer to a base 20 and often use subtraction (Fig. 22).

For example, 80 is ogerin or ogo-erin = 20×4, as in the French (or Danish) "quatre-vingt". Here are some examples of Yoruba number forming, using today's notations:

45 = (20×3) - 10 - 5 50 = (20×3) - 10
108 = (20×6) - 10 - 2 300 = 20×(20 - 5)
318 = 400 - (20×4) - 2
525 = (200×3) - (20×4) + 5

To form the latter, 525, the traditional mathematician proceeded as shown in the illustration: first, 3 "igba" groups of 200 shells each are laid out; next, 4 "ogun" heaps of 20 are removed; finally one "arun" group of 5 is added.

The execution of a multiplication such as 17×19 in the traditional Yoruba way goes as follows (Fig. 23). First, 20 heaps of 20 cowry shells are put in front of the calculator. Next, he takes one shell from each heap, and puts them apart to form a new heap. Next, three heaps of the original 20 heaps are put aside, and from one of them, a shell is added to one of the three heaps,

kan	*1*
meji	*2*
meta	*3*
merin	*4*
maruun	*5*
mefa	*6*
meje	*7*
mejo	*8*
mesan	*9*
mewa	*10*
mokonlaa	*+1+10*
mejilaa	*+2+10*
metalaa	*+3+10*
merinlaa	*+4+10*
meèedogun	*-5+20*
merindinlogun	*-4+20*
metadinlogun	*-3+20*
mejidinlogun	*-2+20*
mokondinlogun	*-1+20*
ogun	*20*
mokonlelogun	*+1+20*
mejilelogun	*+2+20*
metalelogun	*+3+20*
merinlelogun	*+4+20*
meéedogbon	*-5+30*
merindinlogbon	*-4+30*
metadinlogbon	*-3+30*
mejidinlogbon	*-2+30*
mokondinlogbon	*-1+30*
ogbon	*30*

mokonlelogbon	*+1+30*
mejilelogbon	*+2+30*
metalelogbon	*+3+30*
merinlelogbon	*+4+30*
maruundinlogoji	*-5+20×2*
merindinlogoji	*-4+20×2*
metadinlogoji	*-3+20×2*
mejidinlogoji	*-2+20×2*
mokondinlogoji	*-1+20×2*
ogoji	*20×2*
mokonlogoji	*+1+20×2*
mejilogoji	*+1+20×2*
metalogoji	*+1+20×2*
merinlogoji	*+1+20×2*
maruundinlaàadota	*-5-10+20×3*
merindinlaàadota	*-4-10+20×3*
metadinlaàadota	*-3-10+20×3*
mejidinlaàadota	*-2-10+20×3*
mokondinlaàadota	*-1-10+20×3*
àadota	*-10+20×3*
mokonlelaàadota	*+1-10+20×3*
mejilaàadota	*+2-10+20×3*
metalelaàadota	*+3-10+20×3*
merinlelaàadota	*+4-10+20×3*
maruundinlogota	*-5+20×3*

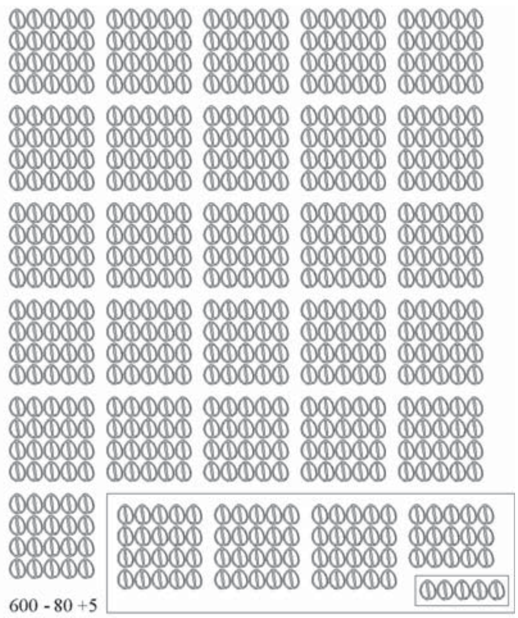


Fig. 22 — Yoruba number names, and the forming of 525.

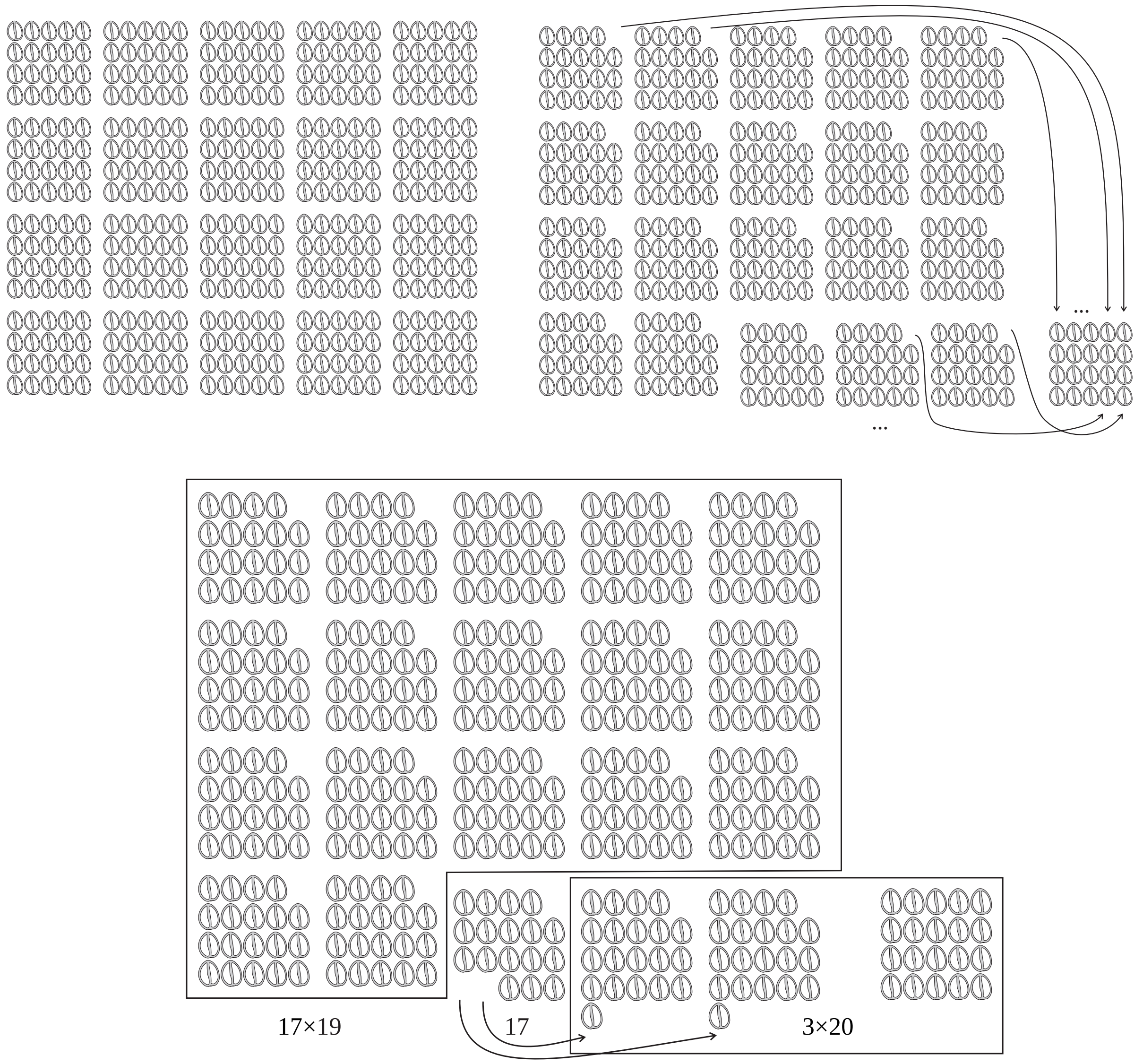


Fig. 23 — The initial situation for the execution of 17×19 (above left), the first step (above right) and the final result (below).

and, again from the same heap, another shell is added to the remaining heap. The two thus completed heaps are appended to the heap formed by putting shells aside from each heap in the first step. Then the Yoruba magician-calculator reads off the result: 17×19 = 323 – the word "magician" is appropriate here.

Funny enough, the English word "score" points to an old use of a counting stick, to sum up to 20 carvings; did England's ancestors use a base 20 too?

6.2. A multiplication method based on doubling and halving

Egyptian arithmetic is well known because of two papyri and secondary Greek and Roman sources. Over 100 mathematical problems reach from approximations for the area of a circle or the volume of a truncate pyramid and of a cylinder (or of a halve sphere – this is subject to discussion), to other standard mathematical problems still taught in school today. Their multiplication method is supposed to have spread from

Greece to Russia, the Middle East and back to Ethiopia, a path so unexpected one can wonder how this can be substantiated. Furthermore, tailors in West Africa would have used it in mental calculations, until recently.

For example, 17×13 goes as follows. The scribe would double the first number, 17 and simultaneously double 1, on and on, to stop until the second number is obtained by a proper addition:

IIII III ∩	I	17	1 /
II ∩ II ∩∩	II	34	2
IIII ∩∩∩ IIII ∩∩∩	II II	68	4 /
IIII∩∩ II ∩ ?	IIII IIII	136	8 /
I∩∩?? ☐	III∩	17+68+136 = 221	1+4+8=13

The symbol for one 1 was I, for 10 ∩, for 100 ?, while the hieroglyph ☐ stood for "the result is the following" and the / sign the numbers to be added.

Because a justified fantasy probably is the best medicine against aversion for mathematics (which may exist among the usual readers of the current publication), we here execute the Ethiopian multiplication method on an African game board. Thus, it is <u>*not*</u> the way this multiplication was executed, but it provides an idea of how to accomplish it without writing. On an igisoro board, that is, a mancala type board with 4 rows of 8 holes, we imagine the holes of each half correspond to 1, 2, 4, 8, 16, 32, 64, 128, and, in the row above, with 256, 512, 1024, 2048, 4096, *etcetera* (Fig. 24). The illustrations show a simple example, 3×6, and a more complicated one, 17×241.

It is not a coincidence some of the numbers correspond to units known from the computer environment: 1024 = one megabyte, 2048 = two megabytes, 4096 = four megabytes, etcetera. N. Wirth, creator of the computer language Pascal, gave an exercise consisting in programming a multiplication of two numbers using only additions, halvings and doublings (Wirth, 1976). Obviously, the comparison of a computer relay and an

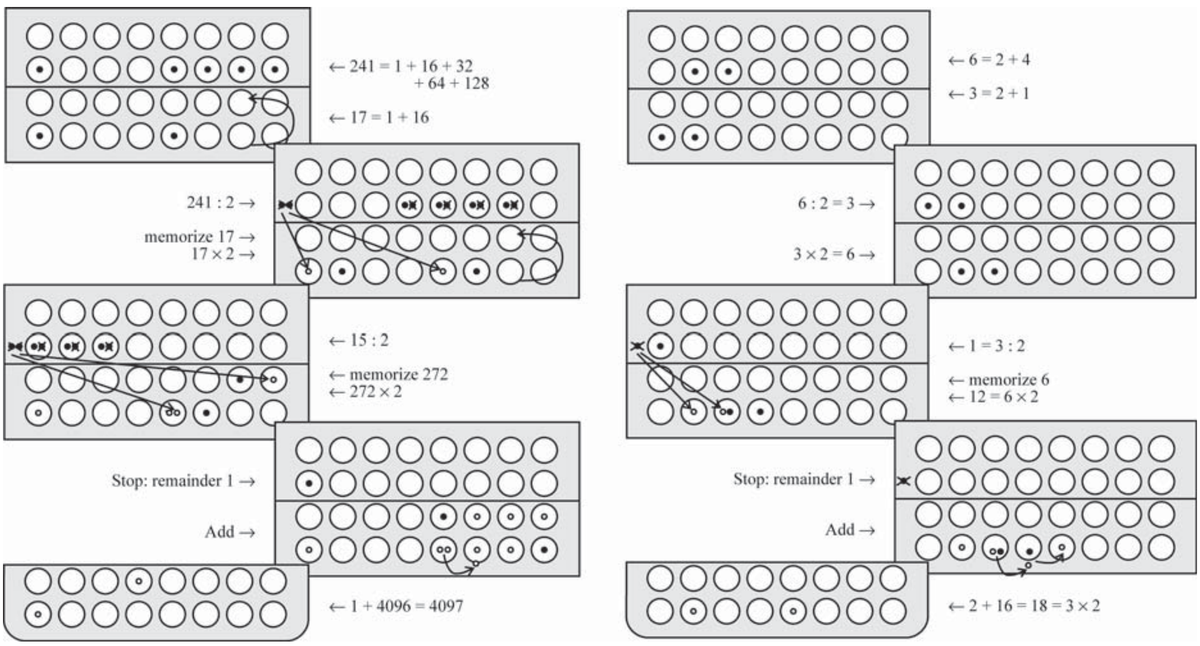


Fig. 24 — 3×6 (left) and 241×17 (right) on an African game board.

African game board is evident: a hole with a pawn is a closed relay, and one without a pawn is an open one. Doubling a number is a “shift”, an instruction known in some programming languages.

6.3. Higher algebra

In small isolated communities, strict rules were sometimes formulated to avoid marriages between close relatives. People belonged to “clans” (this word is not correctly used here, but other alternatives are even more confusion), with, for example, the rule that only people from the same clan are allowed to marry (each other), and that their sons will belong to another clan and their daughters to still another clan, and this, following given prescriptions (Fig. 25 left).

Mathematicians list the numbers of the clans in rows or columns, called “matrices”, to summarize a situation where parents belong to clan 1 (or 2 or 3), their sons to clan 2 (or 3 or 1) and their daughters to clan 3 (or 1 or 2):

$$P(arents) \rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad S(ons) \rightarrow \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \quad D(aughters) \rightarrow \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Here, we will leave these notational questions for what they are, but we can nevertheless provide a nice short example of a mathematical kinship theorem:

Theorem: *in the society described above, a man is allowed to marry the daughter of the brother of his mother.*

Proof.

First, we note a man is always the grandson of his grandparents. If the grandparents belong to clan 1, their daughter, the mother of the man, will belong to clan 3, and so he will be in clan 1. Next, the brother of the mother of the man is the son of the grandparents, and thus belongs to 2, while the daughter of the latter again is in 1. One proceeds in the same way if the grandparents belong to clan 2 or 3: the man and the daughter of the brother of his mother always belong to the same clam and thus are authorized to marry, in that society.

A reader recalling a few facts from the theory of matrices may remember their multiplication, “·”:

$$S \rightarrow \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = S.P$$

and

$$D \rightarrow \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = D.P$$

In this mathematical terminology the proof of the above theorem states that

$$\mathrm{SD} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}_3 = \mathrm{DS}.$$

Mathematicians draw the same conclusion from this as in the above proof in words: “*Quod erat demonstrandum*”.

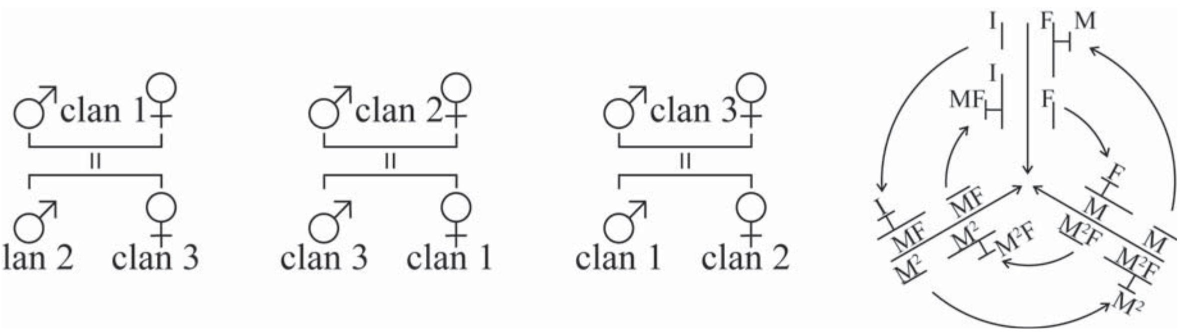


Fig. 25 — A simple example of kinship ties (left) and a traditional representation from the Malekula in Oceania (right). Only the letters are Western additions.

In other societies, there is no limitation to three clans, nor are the rules for marriage identical, and thus this field of study is seminal for many mathematical theorems. For instance, marriages can be monogamous or not, or be permitted with someone from another clan, *etcetera*. Thus, a more general mathematical approach is appropriate, using the notion already mentioned in the paragraph about Crowe's study, that of a "group". In the given example, I_3 (the identity matrix, given above), S and D are said to form the group $\{I_3, S, D\}$. In another example (Fig. 25 right) from Oceania, the group for kinship ties is represented by a more complicated diagram, and mathematicians now use the so-called dihedral group $\{I, M, M^2, F, MF, M^2F\},\cdot$ of order 6. In the present context it does not matter much what this notion may stand for, but this Warlpiri-type example is particularly interesting, as the Warlpiri are said to have no knowledge of number at all, lacking number words even for the smallest quantities, and still, their kinship relations provide an interesting mathematical topic.

For many examples from Oceania, Burkina Faso and southern Africa, this group structure is the adequate algebraic translation, but in the larger African context, an even more general description for kinship relations is necessary, using the so-called "semi-group" notion, such as for the Baoulé, who do not allow two brothers to marry two sisters.

A few years ago, Alain Gottcheiner defended his D. E. A. in mathematics at the "Université Libre the Bruxelles" (Gottcheiner 2001), with Prof. Em. F. Buekenhout as promoter. The mathematical approach has the advantage of uniting all kinds of statements used by anthropologists' jargon in a single overview. Although this ethno-modeling process is subject to critique, a tasteful endnote is provided by the Belgian philosopher Leo Apostel, quoted in a kinship PhD of yet another scholar, Tjon Sie Fat (1990):

It is the aim to extend the analysis of Lévi-Strauss, indicated by T, so that different hybrid systems can be included. Here, the letter T refers to the ordered foursome R(S, P, M, T), in which Apostel summarized the important variables of a modeling process […]. Thus, the purpose of the mathematical study of the family ties structure can be considered as a restructuring system or a system extension (Apostel, 1961).

The latter studies show the presented ethnomathematics are not merely "recreational", for sure, and some of the "illustrations", such as figure 25, need special examination to be understood.

The Ishango object itself has not been discussed in detail here, as this was, is and will be done at large in other papers (or, see Brooks *et al.*, 1995; de Heinzelin de Braucourt, 1957, 1962 ; Marshack, 1972; Pletser, 1999, 2000; Post Office, 2000). The author is also aware of the fact the given examples of "African" mathematics range over many regions and over a large period of time, and the earliest examples may rather belong to what is sometimes called "proto-mathematics". The use of this word would avoid unnecessary polemics as to decide whether the Ishango carvings are of mathematical nature or not. Nevertheless, to Prof. Em. F. Beukenhout, member of the Belgian Academy, even this matter is pointless: *after all, wasn't the very mathematician a woman, called Lucy?* (Beukenhout, 2000)

7. NOTE

The author wishes express his gratitude towards the "Société royale belge d'Anthropologie et de Préhistoire – Koninklijke Belgische Vereniging voor Antropologie en Prehistorie" for the open-minded collaboration which led to the current paper. As for the illustrations, the help of the Royal Museum for Central Africa of Tervuren (BELGIUM) and of the ADIA group at the Royal Belgian Institute of Natural Sciences of Brussels (BELGIUM) was crucial. Interdisciplinary research between mathematicians, archaeologists and anthropologists is not only a matter of the past; it also has a future.

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